1-D Signal Prediction Using Wavelets.

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Abstract: *This paper presents a novel methodology for improving 1-D signal prediction performance of MultiLayer Perceptrons (MLPs) by incorporating wavelet based features in their input vectors. In addition, the wavelet basis providing the best such input features is extensively experimentally investigated. To this end, the generalization and learning convergence properties of MLPs trained with patterns extracted by involving the 1-D DWT transform with Haar, Daubechies, Symmlet, Coiflet and Vaidyanathan bases are compared, when these MLPs are applied to 1-D irregular signals. It is shown that, although all the above wavelet bases result in the extraction of input features leading to the design of MLPs with enhanced signal prediction properties, 1-D Daubechies DWT demonstrates the best performance.*
Keywords : *signal modeling, wavelets, multriresolution analysis, signal prediction.*

1. INTRODUCTION

Information processing of signals has become an essential part of contemporary scientific and developmental activity in the field of computer science and engineering. It is used in telecommunications, in diagnostic systems, in the transmission and analysis of biomedical signals and in time series forecasting among many important disciplines of computer aided engineering. Signal prediction in particular is one of its major targets that plays an integral role in numerous applications. For instance, medical diagnosis involves biomedical signal modeling aiming at extracting important information that characterizes a patient, telecommunications and data transmission employ signal modeling to achieve effective data compression and time series forecasting needs signal prediction of fluctuations in time. All these applications involve the analysis and interpretation of complex time series which, in general, can be characterized and represented by non smooth and highly irregular signals. The goal of this paper can be summarized as an attempt to achieve robust prediction of signals with irregularities by suggesting a novel methodology involving neural networks and the wavelet signal decomposition. Moreover, it mainly aims at experimentally finding out the wavelet basis leading to best prediction results. The various methodologies, which have, so far, been developed for accomplishing successful signal representation, mainly come from linear system theory [1], stochastic process theory [2], the black-box methodology [3] and dynamical systems analysis [4]. The approach adopted here is based on the black-box methodology and more specifically on neural networks function approximation techniques of the MLP type. It has been proved that these feedforward neural networks are universal function approximators for any continuous and regular function [5]. It is demonstrated here that the efficiency of such powerful continuous function prediction techniques can be significantly increased when attempting to represent highly irregular functions (signals) by using the wavelet analysis tools. Thus, the main contribution of this paper is the experimental illustration of the fact that the wavelet signal decomposition can provide the means for extending the function approximation capabilities of MLP in the case of irregular functions independently of the wavelet basis chosen.

2. METHODOLOGY AND RESULTS

The most important point in the suggested methodology is the fusion of the original signal data and the coefficients of their corresponding DWT (Discrete Wavelet Transform) multiresolution analysis for achieving improved solutions through treating these data as input patterns to the MLP function approximation module. While contemporary time domain signal prediction methods involve present along with past sampled signal values as input features, new additional informative features are actively sought here in the wavelet domain. The optimal features extracted from this domain and herein proposed for improved prediction of irregular signals are the detail wavelet coefficients associated with the channel corresponding to the high-pass filtered version of the original signal. There is no, so far, attempt in the literature for investigating the use of the wavelet coefficients as input patterns in function approximation techniques for signal prediction problems.

The function approximation module of the suggested approach utilizes neural networks of the MLP type for predicting signals with abnormal fluctuations. These feedforward networks are trained and tested using the patterns extracted by employing the procedure analyzed next. A sliding window of length M scans all the N samples of the original signal. For the M signal samples corresponding to its current position, their 1-D DWT transform is considered with one out of the several different wavelet bases involved in this paper. The associated M/2 detail wavelet coefficients are afterwards extracted. After normalization in the [0,1] interval, these M + M/2 real numbers (the M signal samples and their M/2 detail wavelet coefficients) construct the MLP input vectors. The two future signal values corresponding to the same sliding window comprise the desired response vector for this MLP input vector.

We have conducted an extensive experimental investigation in order to demonstrate the efficiency of our signal prediction methodology in improving the function approximation performance of MLP. We have attempted to apply our approach to the highly irregular signal shown in figure 1. This signal is not artificial. It has been extracted, instead, from a 1024 X 1024 image representing a complex scene. It is, actually, one randomly selected row of the matrix corresponding to this image. Therefore, 1024 samples of the associated signal of figure 1 have been considered for further processing. Following the above described procedure 991 in total patterns have been obtained, by applying a sliding window of length 32 to the original signal samples. Based on these patterns, we have organized an extensive evaluation of several methodologies for improving the signal prediction performance of MLPs applied to the signal of figure 1. More specifically, we herein compare:

1) an MLP 32-35-35-2, whose input vector's components are the 32 signal samples corresponding to the sliding window scanning the signal
2) an MLP 48-35-35-2, whose input vector's components include the 32 signal samples corresponding to the sliding window scanning the signal as well as the 16 detail wavelet coefficients of their associated 1-D Haar DWT.
3) an MLP 48-35-35-2, whose input vector's components include the 32 signal samples corresponding to the sliding window scanning the signal as well as the 16 detail wavelet coefficients of their associated 1-D Coiflet (filter length 5) DWT.
4) an MLP 48-35-35-2, whose input vector's components include the 32 signal samples corresponding to the sliding window scanning the signal as well as the 16 detail wavelet coefficients of their associated 1-D Daubechies (filter length 12) DWT.
5) an MLP 48-35-35-2, whose input vector's components include the 32 signal samples corresponding to the sliding window scanning the signal as well as the 16 detail wavelet coefficients of their associated 1-D Symmlet (filter length 10) DWT.
6) an MLP 48-35-35-2, whose input vector's components include the 32 signal samples corresponding to the sliding window scanning the signal as well as the 16 detail wavelet coefficients of their associated 1-D Vaidyanathan DWT.

The comparison has been carried out in a pairwise fashion between each one of the methods 2-6, on the one hand, and method 1. Each one of the MLPs involved has been trained with the 90% of the patterns randomly selected from the ones obtained after applying the associated method to the initial 991 signal patterns. It has, then, been tested with all its corresponding 991 patterns. The On-line Backpropagation error algorithm has been employed after finding out the optimal learning rate and momentum coefficients for each case (for all methods, after a lot of experiments, these coefficients are 0.2 and 0.3 respectively). The results of the application of all these methodologies are shown in figures 3-12 and table 1. Figure 2 shows figure-1's signal reconstruction when method 2 has been applied.

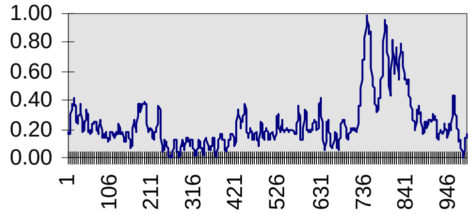

Figure 1. Original signal

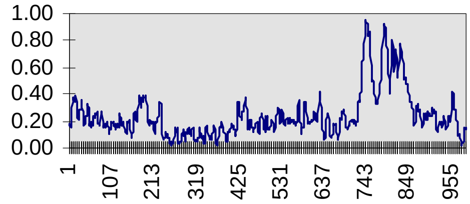

Figure 2. Reconstructed signal of Fig. 1 using the proposed method 2.

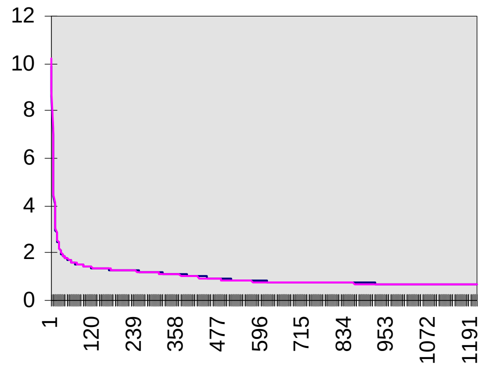

Figure 3. Learning Error Curves, Methods 2 (best curve) and 1 (worst curve).

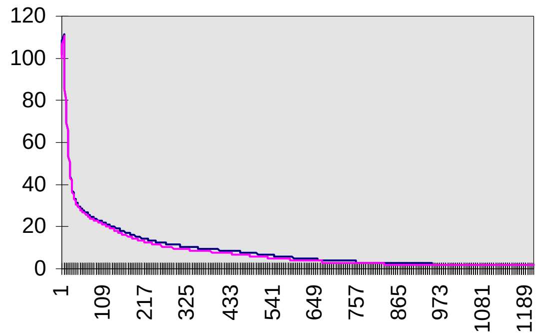

Figure 4. Generalization Error Curves, Methods 2 (best curve) and 1 (worst curve).

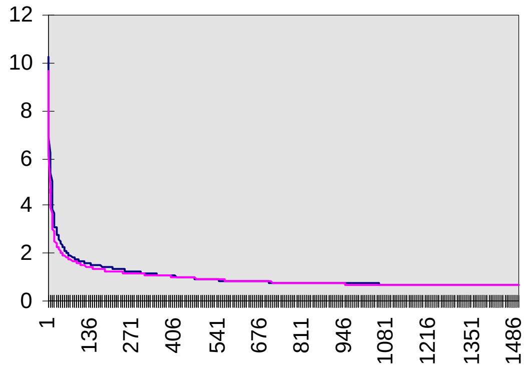

Figure 5. Learning Error Curves, Methods 3 (best curve) and 1 (worst curve).

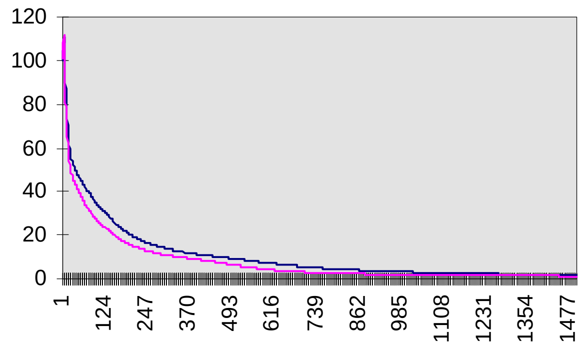

Figure 6. Generalization Error Curves, Methods 3 (best curve) and 1 (worst curve).

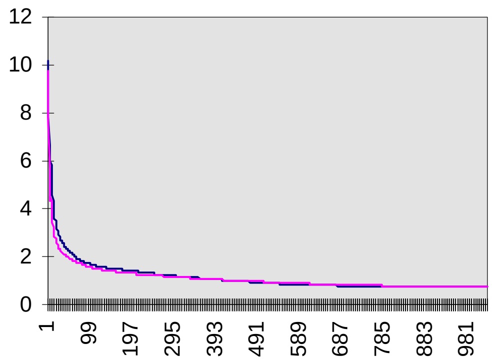

Figure 7. Learning Error Curves, Methods 4 (best curve) and 1 (worst curve).

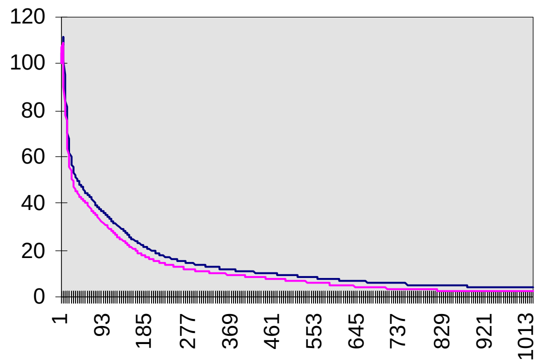

Figure 8. Generalization Error Curves, Methods 4 (best curve) and 1 (worst curve).

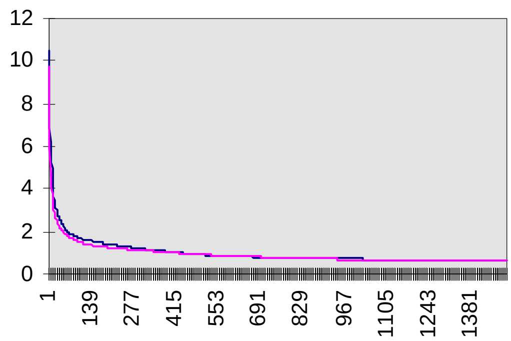

Figure 9. Learning Error Curves, Methods 5 (best curve) and 1 (worst curve).

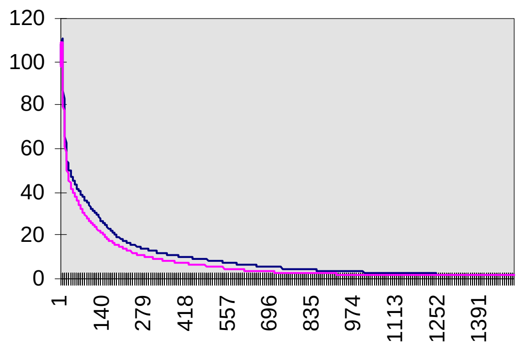

Figure 10. Generalization Error Curves, Methods 5 (best curve) and 1 (worst curve).

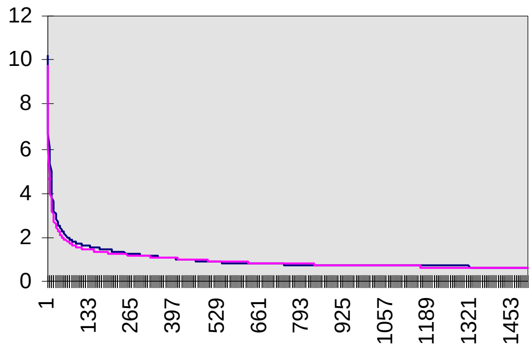

Figure 11. Learning Error Curves, Methods 6 (best curve) and 1 (worst curve).

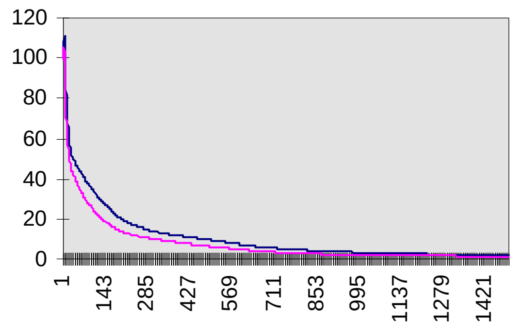

Figure 12. Generalisation Error Curves, Methods 6 (best curve) and 1 (worst curve).

Method	Generalization error (SSE) for MLPs of method 1	Generalization error (SSE) for MLPs of the wavelet methods	Percentage improvement for the 2 previous columns
2	1.68	1.53	9%
3	1.85	1.35	27%
4	**3.94**	**2.33**	**41%**
5	1.79	1.5	17%
6	1.95	1.32	33%

Table 1. Signal prediction performance results for the compared methods

3. DISCUSSION AND PROSPECTS

A new robust method for prediction of irregular signals has been presented using the wavelet transform for improved feature extraction. More specifically, the initial vectors containing the signal samples organized in sliding windows, are augmented with suitably selected wavelet coefficients of the 1-D DWT applied with several different bases. In particular, the 1-D Haar, Daubechies, Symmlet, Vaidyanathan and Coiflet DWTs have been considered. It is extensively experimentally demonstrated that when the obtained vectors are used as input vectors of function approximators, like MLPs, the result is a significant improvement in both their training and generalization performance, despite the increase in input space dimensionality and independently of the wavelet basis involved. The 1-D Daubechies DWT, however, it is shown to give the best results, at least in the experiments attempted. Extensive investigations of our methodology for its applicability in financial time series forecasting are now in a preliminary stage of development.

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